

# Process Machine Interactions

Ichiro Inasaki

Chubu University, Japan

E-mail: inasaki@isc.chubu.ac.jp

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## Abstract

The aim of this paper is to provide some guidelines to achieve high performance machining represented by cutting and grinding shedding light on the process machine interactions. Among various kinds of interactions, an interaction mediated by machining force and elastic deformation of the machine tool system will be discussed. In addition, measures to attain higher machining accuracy with the aid of ultrasonic vibration and to increase the stability limit of the process will be described.

Keywords: Cutting, Grinding, Accuracy, Productivity, Force, Deformation, Instability

## 1 INTRODUCTION

Two ultimate goals of the machining process represented by cutting and grinding are high accuracy and high productivity. The machining force generated during the material removal process causes elastic deformations in the machine tool system which includes the workpiece and the cutting tool. In addition, it may cause dynamic instability of the machining process resulting in machining errors and a decrease in productivity.

In order to meet requirements of high accuracy and productivity, it is necessary to understand well the interaction between the process and the machine tool system linked with the force and the deformation. In this paper, models for process machine interactions will be presented taking examples of cutting and grinding.

## 2 STATIC INTERACTION

In cutting and grinding, the process and the machine tool system constitute a closed loop linked with the force and the deformation as illustrated in Figure 1. In order to enhance the process performance, it is essential to understand well the characteristics of each element and the interaction of these comprising elements. To predict the resultant process performance, the physical models of the mechanical system as well as the process should be obtained.

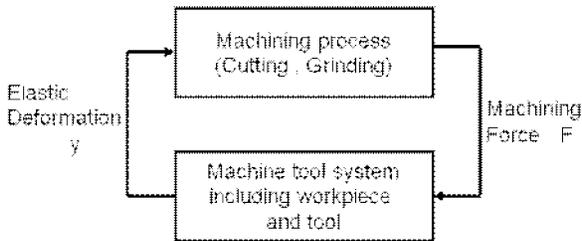


Figure 1: Machining process as a closed loop system.

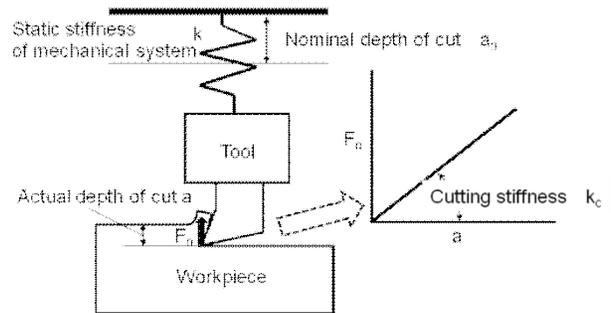
### 2.1 Cutting Process

Let us take the example of the simple orthogonal cutting process illustrated in Figure 2. The thrust component of the resultant cutting force is given by the following equation <sup>1)</sup>:

$$F_n = ba\tau_{cr}(\cot\phi \tan(\phi + \beta - \gamma) - 1) \quad (1)$$

where  $b$  is the width of cut,  $a$  is the undeformed chip thickness,  $\tau_{cr}$  is the shear strength of the work material,  $\phi$  is the shear angle,  $\beta$  is the friction angle between the chip and the rake face of the cutting tool, and  $\gamma$  is the rake angle of the cutting tool. We can rewrite Equation (1) as follows:

$$F_n = k_c a \quad (2)$$



where

$$k_c = b\tau_{cr}(\cot\phi \tan(\phi + \beta - \gamma) - 1) \quad (3)$$

Figure 2: Model of orthogonal cutting process.

The unit of  $k_c$  in Equation (2) is force / length, i.e., the stiffness. Therefore, let us call  $k_c$  the "cutting stiffness". In other words, we can assume that there is a spring having the stiffness  $k_c$  between the workpiece and the cutting tool. Equation (2) is a physical model for the cutting process.

The physical model for the machine tool system is given by the following equation:

$$y = \frac{F_n}{k} \quad (4)$$

where  $y$  is the elastic deformation and  $k$  is the static stiffness of the machine tool system including the workpiece and the cutting tool. Assuming that the nominal depth of cut is  $a_n$ , the following relationship is obtained:

$$a = a_n - y \quad (5)$$

From equations (2), (4) and (5), we can derive the following result:

$$\frac{a}{a_n} = \frac{1}{1 + \frac{k_c}{k}} \quad (6)$$

If the static stiffness of the machine tool system  $k$  is infinitely large,  $a=a_n$ , therefore, the machining error does not occur. However, this is not the case for the practical cutting process. Consequently, the larger the mechanical stiffness is, and the smaller the cutting stiffness is, the smaller is the machining error. The ratio between the cutting stiffness and the machine tool system, i.e., the process machine interaction has a decisive influence on machining accuracy.

Taking the above described discussion into account, the cutting process can be represented by the block diagram as shown in Figure 3. The cutting stiffness and the static compliance (inverse of the static stiffness) of the machine tool system can be regarded as transfer functions in the cutting system. The system has one feed-back loop linked with the elastic deformation of the mechanical system.

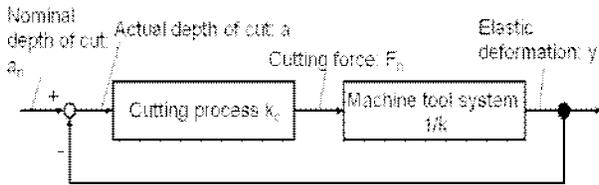


Figure 3: Static cutting process as a closed loop system.

## 2.2 Grinding Process

Even today, the grinding process remains as a critical material removal process for achieving high machining accuracy. Figure 4 shows a cylindrical plunge grinding system, where  $x=v_f t$  is the grinding wheel travel ( $v_f$ : in feed rate of the grinding wheel head stock,  $t$ : time). Assuming that the elastic deformation of the grinding machine system caused by the normal force component of the grinding force  $F_n$  is  $y$  and the reduction of the workpiece radius is  $r$ , the following relationship is obtained:

$$r = x - y \quad (7)$$

If the mechanical system has the static stiffness  $k$ , the elastic deformation generated in the mechanical system is given by Equation (4). Therefore,

$$r + \frac{F_n}{k} = v_f t \quad (8)$$

Assuming that grinding power  $P=F_t V$  ( $F_t$ : the tangential grinding force,  $V$ : the grinding wheel peripheral velocity) is in proportion to the material removal rate  $Q$ , the normal component of the grinding force  $F_n$  is given by

$$F_n = \kappa b \pi d_w \frac{1}{V} \frac{dr}{dt} \quad (9)$$

where  $\kappa$  is a constant,  $b$  is the grinding width and  $d_w$  is the workpiece diameter. Equation (9) is a model equation for the grinding process. From above relationships, the following differential equation is obtained.

$$r + \frac{\kappa b \pi d_w}{k} \frac{1}{V} \frac{dr}{dt} = v_f t \quad (10)$$

Equation (10) shows that the cylindrical plunge grinding process is found to be the first order time delay system. Therefore, its solution will be

$$r(t) = v_f t - v_f T (1 - e^{-\frac{t}{T}}) \quad (11)$$

where  $T$  represents the time constant of the first order time delay system which is given as<sup>2)</sup>:

$$T = \frac{\kappa b \pi d_w}{k V} \quad (12)$$

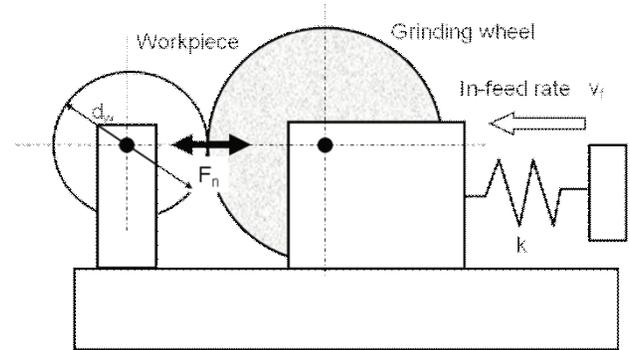


Figure 4: Cylindrical plunge grinding process.

The time constant represents the interaction between the process and the machine tool system.

Equation (11) is graphically represented in Figure 5 for the grinding time using the time constant as a parameter. The grinding cycle is supposed here to be comprised of a constant in-feed rate and spark-out grinding after stopping the in-feed motion. The solid line in the diagram represents the movement of the wheel spindle stock, whereas the broken lines indicate actual reductions of the workpiece radius. The difference between the wheel travel and the workpiece radius change causes machining errors. From Figure 5, it can be pointed out that the lower the mechanical stiffness is, the larger such difference will be, in other words, the process-machine interaction has a decisive influence on whole process behaviour.

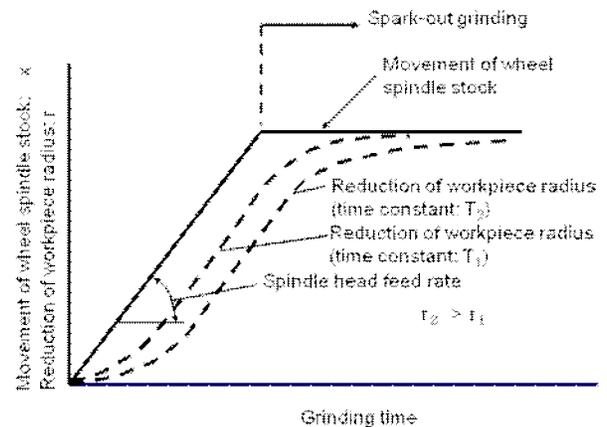


Figure 5: Grinding process as the first order time delay system.

In the grinding process, the elastic deformation of the grinding wheel cannot be ignored and must be taken into account in the resultant deformation of the mechanical system.

## 2.3 Reduction of the Cutting Stiffness with the Aid of Ultrasonic Vibration

There are some conceivable measures for achieving high accuracy machining: to increase the static stiffness of the

machine tool system and to decrease the cutting stiffness as indicated by Equation (6). One of the possible and practical means to reduce the cutting stiffness is an application of the ultrasonic vibration to the cutting process<sup>3</sup>. Figure 6 shows the principle of the ultrasonic vibration cutting, where the vibration is applied in the direction of the cutting velocity.

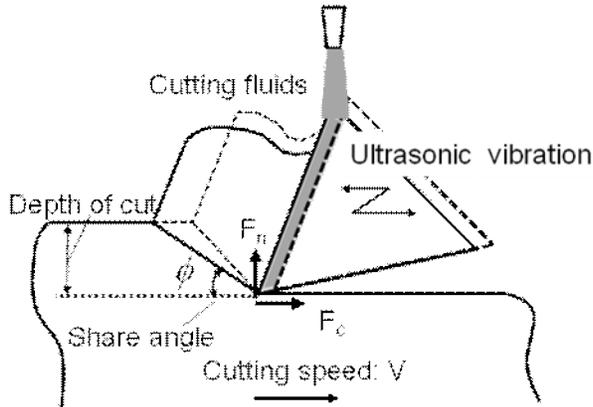


Figure 6: Effect of ultrasonic vibration in cutting.

Figure 7 shows significant effects of vibration cutting for reducing the cutting force, i.e., the reduction of the cutting stiffness. According to Equation (6), the residual depth of cut can be significantly reduced and consequently the machining errors can be suppressed. This method is particularly advantageous for cutting of slender workpieces having low stiffness, because it makes it possible to eliminate the use of an additional device to support the workpiece.

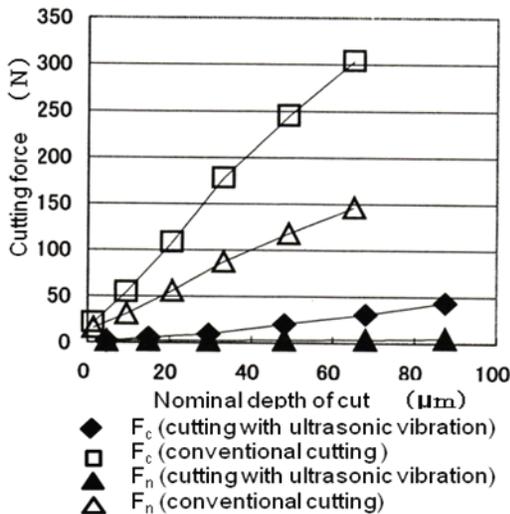


Figure 7: Reduction of cutting force by ultrasonic vibration.

The reason why the cutting force reduces through the application of ultrasonic vibration is considered due to a decrease in the friction coefficient between the chip and the rake face of the cutting edge. Therefore, it should be mentioned here that the application of ultrasonic vibration

is effective for reducing the cutting force only when the following condition is satisfied:

$$V < 2\pi af \quad (13)$$

where  $V$  is the cutting speed,  $a$  is the amplitude of the ultrasonic vibration, and  $f$  is its frequency. The friction coefficient cannot be reduced as long as the contact between the chip and the rake face of the tool edge is kept during cutting.

### 3 DYNAMIC INTERACTION

#### 3.1 Vibration Phenomena in Machining

In order that high accuracy and high productivity be maintained in the machining process, no vibrations should be allowed during the process. Degradation of the workpiece contour and roughness due to the vibration in grinding can be more severe than that caused by the vibration in cutting. The vibration generated in machining is classified into two kinds by cause: forced vibration, and self-excited vibration. The forced vibration is that which is generated when there exists a vibration source that oscillates the mechanical structure. For example, imbalance of the rotating parts in machine tools and the fluctuating cutting force during intermittent cutting, such as milling, represent the most serious sources of the vibration. The forced vibration of the latter type is a typical process machine interaction.

While the amplitude  $y$  of the vibration generated by exciting force  $F$  is a function of the excitation frequency, it becomes largest when the excitation frequency coincides with the natural frequency of the mechanical system, i.e., at resonance. The maximum amplitude is expressed as follows<sup>4</sup>:

$$\frac{y}{F} = \frac{1}{2k\zeta} \quad (14)$$

where  $\zeta$  represents the damping ratio. To increase the dynamic stiffness, it is important to improve not only the static stiffness  $k$ , but also the damping ability.

Another vibration source is the self-excited vibration due to the regenerative effect. This is an unstable vibration based on a time delay phenomenon. Supposing that a relative displacement is generated between the cutting tool and the workpiece for one reason or another in cutting, its effect will be left on the workpiece surface in the form of cyclic waviness. When the same affected area on the workpiece comes back to cutting after one revolution of the workpiece in the case of turning, its effect shows up as a change in the depth of cut or undeformed chip thickness. This unstable phenomenon is called regenerative effect. As a matter of course, this problem can happen in the most of cutting and grinding. In the case of grinding, the grinding wheel may also be susceptible to cyclic waviness generated through the vibration and wear. In this way, the vibration phenomena can be exceptionally involved due to existence of the regenerative effect on the grinding wheel surface making the vibration phenomenon more complex.

Occurrence of the self-excited vibration deteriorates machining accuracy as well as productivity. This is due to the fact, that in order to suppress the vibration, the depth of cut, i.e., the material removal rate, should be decreased.

### 3.2 Stability limit

The dynamic cutting process can be represented by the block diagram having two feedback loops as shown in Figure 8. In addition to the primary feedback loop shown in Figure 3, the second feedback loop, the regenerative feedback loop should be taken into account for the dynamic cutting process<sup>5</sup>.

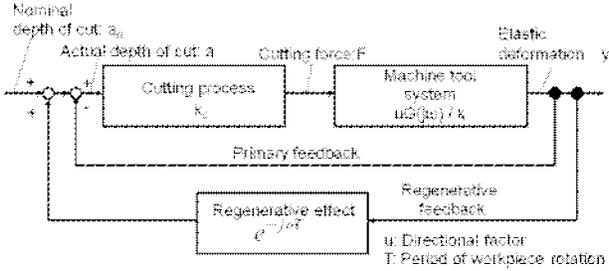
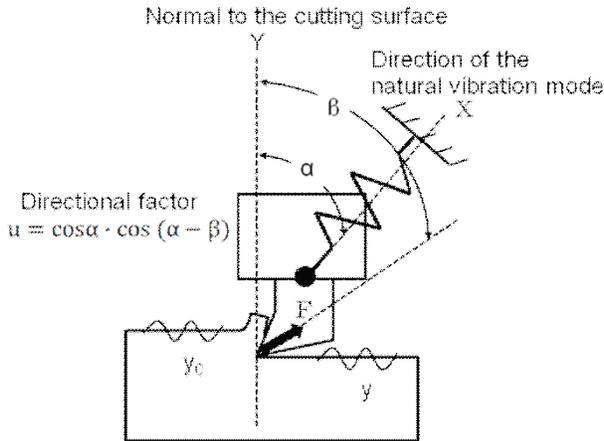


Figure 8: Dynamic cutting process.

Assuming the parting process using a turning machine, let us derive the stability limit of the process. Relative vibration in the direction Y (normal to the cut surface) causes a waviness  $y_0$  on the workpiece surface. In the subsequent cut, vibration occurs with the amplitude  $y$  in the direction Y. Let us assume that the vibratory system which consists of the machine tool, the cutting tool, and the workpiece, is the single degree of freedom system (direction of the natural vibration mode is X) such as that illustrated in Figure 9, where  $F$  is the dynamic cutting force. Assuming that the cutting force is proportional to the undeformed chip thickness (Equation(2)), the amplitude of the dynamic cutting force is given by



$$F = k_c(y_0 - y) \quad (15)$$

Figure 9: Cutting system of a single degree of freedom.

The vibration amplitude  $y$  in the direction Y oscillated by the dynamic cutting force is:

$$\frac{y}{F} = \frac{u}{k} G(j\omega) \quad (16)$$

where  $k$  is the static stiffness of the machine tool system,  $G(j\omega)$  is its non-dimensional direct compliance, and  $u$  is the directional factor given by

$$u = \cos\alpha \cdot \cos(\alpha - \beta) \quad (17)$$

$\alpha$  and  $\beta$  in Equation (17) are the angle between (Y) and (X) and the angle between (Y) and (F), respectively.

From Equations (15) and (16),

$$\frac{y_0}{y} = \frac{\frac{u}{k_c} + \frac{u}{k_c} G(j\omega)}{\frac{u}{k_c} G(j\omega)} \quad (18)$$

Equation (18) describes the closed-loop system of self-excited vibration. The condition for the limit of stability is expressed as:

$$\left| \frac{y_0}{y} \right| = \left| \frac{\frac{u}{k_c} + \frac{u}{k_c} G(j\omega)}{\frac{u}{k_c} G(j\omega)} \right| = 1 \quad (19)$$

which signifies that the amplitude  $y$  of vibration in a cut will be equal to the amplitude  $y_0$  of vibration in the preceding cut. The system is stable when  $|y_0 / y| > 1$ , and unstable when  $|y_0 / y| < 1$ . Condition (19) can be satisfied only if<sup>6</sup>:

$$-\frac{1}{2k_c} = \frac{u}{k} \text{Re}G(j\omega) \quad (20)$$

where  $\text{Re}G(j\omega)$  is the real part of the direct compliance of the machine tool system. Equation (20) is the simplest form of the condition for the limit of stability. The stability of the cutting process can be discussed by using Figure 10, which illustrates the vector locus of the machine tool system and the cutting process. The condition when the both locus intersect each other is the stability limit. Therefore, the critical limit of stability condition can be expressed as:

$$-\frac{1}{2k_c} = \frac{u}{k} [\text{Re}G(\omega)]_{\min} \quad (21)$$

where,  $[\text{Re}G(\omega)]_{\min}$  is the maximum negative real part of the direct compliance. This is given by

$$[\text{Re}G(j\omega)]_{\min} = \frac{-1}{4\zeta(1+\zeta)} \quad (22)$$

where,  $\zeta$  is the damping ratio of the mechanical system.

Equation (22) can be obtained through

$$\frac{d[\text{Re}G(\omega)]}{d\omega} = 0 \quad (23)$$

From equations (21) and (22), the critical limit of stability for the orthogonal cutting process is

$$\frac{1}{k_c} = \frac{u}{2k\zeta(1+\zeta)} \quad (24)$$

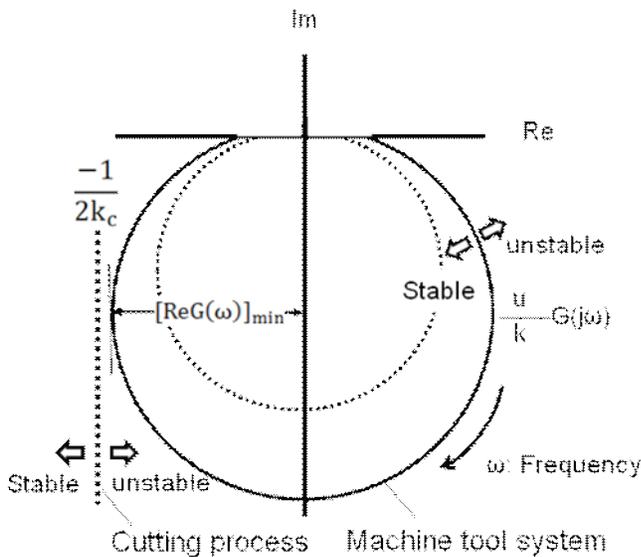


Figure 10: Vector locus of dynamic cutting process.

In order to make heavy duty cutting (cutting with a large cutting stiffness  $k_c$ ) possible, the machine tool system must have the high static stiffness  $k$  and the damping ratio  $\zeta$ . In addition, small value of the directional factor  $u$  is effective to achieve better stability. Therefore, Equation (24) indicates the close interaction between the process and the machine tool system.

### 3.3 Directional Factor

As described in the previous chapter, the directional factor has a significant influence on the stability. Definition of the directional factor given by Equation (17) indicates the typical process machine interaction in terms of the directions of the cutting force, the vibration mode, and the depth of cut. The increase of static stiffness to achieve higher stability results in the machine tool structure having large size and heavy weight. Paying attention to the directional factor, there is a possibility to design machine tools with light weight structure.

The influence of the directional factor on the stability limit is given in Figure 11 for a single degree of freedom system. It is a polar diagram with the angle  $\alpha$  as the angular coordinate and with the cutting stiffness  $k_c$  plotted on the radius. For two directions given by  $\alpha = \pi/2$  and  $\alpha = (\pi/2) + \beta$ ,  $k_c$  is infinitely large. This means that the process is absolutely stable. Because of the multiple vibration modes of practical machine tool systems, the infinite stability can never be attained. However, it is worthwhile to consider the influence of the directional factor on the stability at the design stage of the machine tool structure.

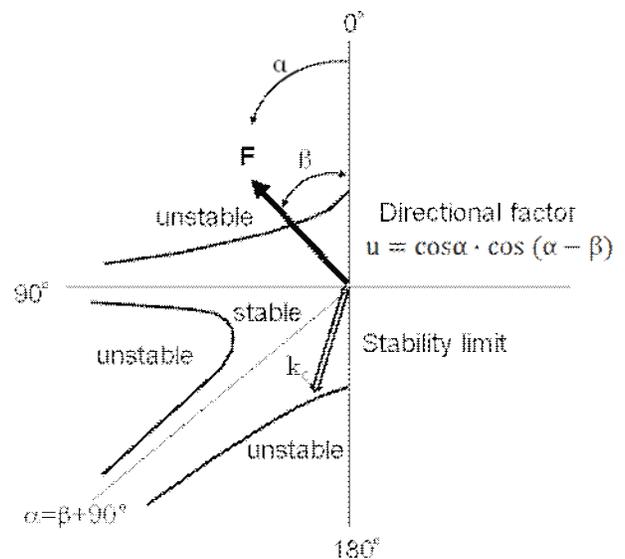


Figure 11: Influence of directional factor on stability.

### 3.4 Measures to Improve Dynamic Stability

The generation of self-excited vibration does not only degrade machining accuracy, but also leads to decline of productivity. Therefore, it is critical to suppress such vibration to achieve high performance cutting. Vibration suppressing measures can be achieved only by considering the behavior of the process and the machine tool system as well as the interaction thereof.

Some concepts for suppressing the self-excited vibration are also shown in Figure 10. A broken straight line drawn parallel to the imaginary axis in the negative real section represents the cutting process, which moves close to the imaginary axis when the cutting stiffness increases. Another vector locus indicates vibration characteristics of the machine tool system. The result of stability analysis demonstrates that the machining system becomes unstable with generation of self-excited vibration when the straight line has intersecting points with the vector locus of the machine tool system. Therefore, vibration suppressing measures should be such that these intersections do not happen. Specifically, the following countermeasures which can be applied to the grinding process as well are derived <sup>7)</sup>:

- (1) Change the cutting conditions in such a way that moves the line parallel to the imaginary axis leftward. In many such cases, however, a decline of the productivity may result.
- (2) Increase the static stiffness of the mechanical system or reduce the directional factor so as to minimize the maximum stand-out of the vector locus of the machine tool system in the negative real section <sup>8)</sup>.
- (3) Increase the damping ratio of the machine tool system.
- (4) Move the entire vector locus rightwards, which can be achieved to a certain extent by introducing an elastic element in the vicinity of the cutting tool edge.
- (5) Disturb the regenerative effects by staggering the phase shift between the vibration  $y$  and the undulation on the workpiece surface  $y_0$ . This can be achieved by changing the rotational speed of the workpiece in turning and the cutting tool in milling,

respectively <sup>9)</sup>.

#### 4 CONCLUSIONS

With a view to gain guidelines for achieving high performance machining in terms of accuracy and productivity, we have pointed out that it is essential to grasp the behavior of the process and the machine tool system as well as to understand the interaction thereof<sup>10)</sup>. As far as the material removal process is concerned, first of all it is necessary to obtain a reliable physical model that explains how the machining force changes depending on the process condition, and then to search for deformation response of the machine tool system against the static and dynamic machining forces.

In the practical machining processes, in addition to the deformation caused by the force, the deformation due to the heat generation should be also taken into account.

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